Model formalisms, continued; CA, MAPs, ODE, Event-based

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Yesterday

- Computational Biology: here studying multilevel informatic processes with muliple dynamical models
- understanding counter intuitive micro-meso-macro transitions through such modeling
- model requirements: *unique next state function*
  FSM $->$ (multiple) attractors ($+$ Garden of Eden states)
- model formalisms as constraints
  FSM subsystems: CA
  – very simple rules $->$ maximally complex behavior
  – Mesoscale patterns – Zoo

QUESTIONS?

TODAY

- CA as modeling tool: examples
- alternative constraint (shortcut): ordering of states ODE
- CA , ODE, MAP's EVeNTS as dynamical systems: common/non-common concepts/features.
- Individual based models

CA as ....

prototype local interactions give complex behavior

- dynamical system
- experimental mathematics , artificial physics(Ulam)
- artificial life (von Neumann, Langton)
- 'new' physiscs (Wolfram):

  ....Universe is 3D CA *"we only have to find the transition table"*
- **modeling tool** *particle based* (Toffoli)
- (NOT bad PDF)

Paradigm system

generic behavior, counter example - existence proof

CA as modeling tool: states vs particles/individuals/alleles

spread of neutral genes vs diffusion in CA
global vs local particle conservation

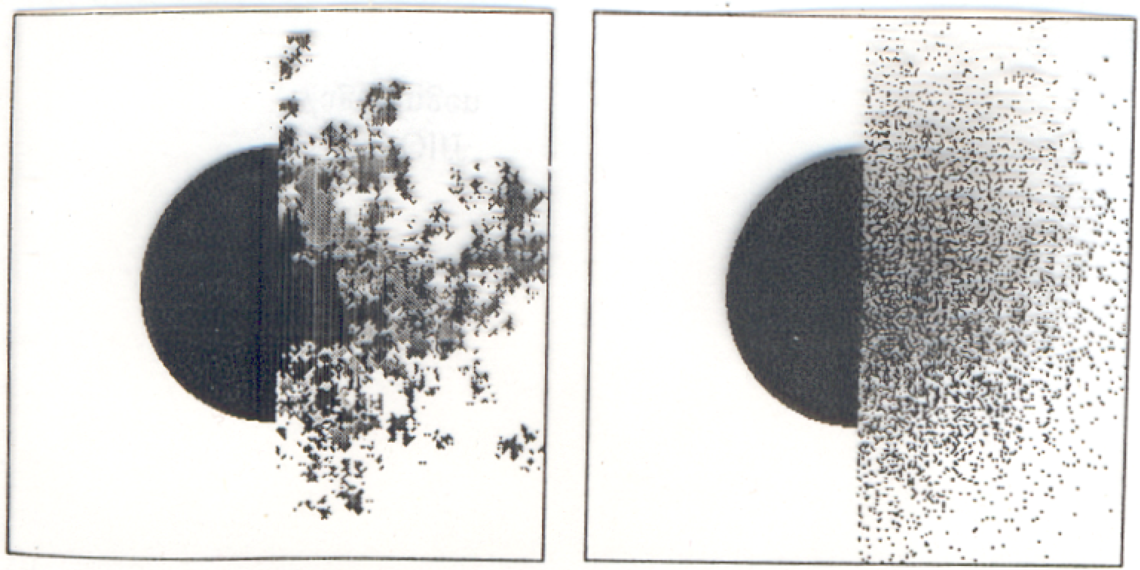
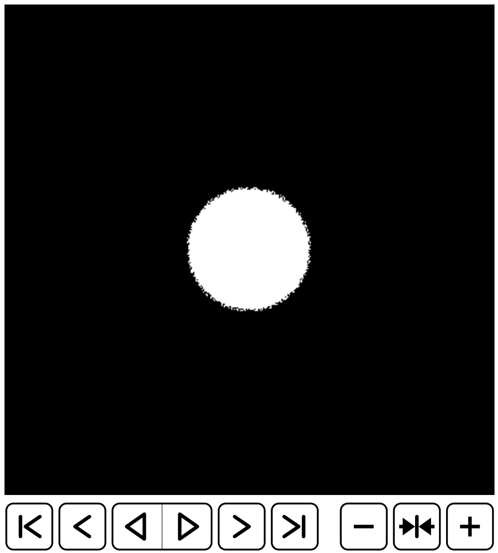


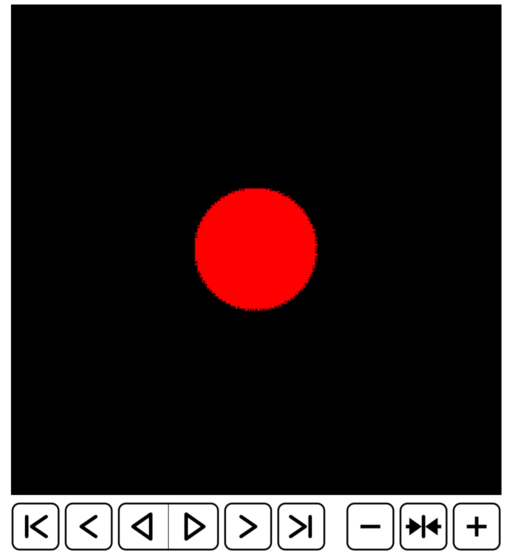
Figure 9.3: (a) Pseudo-diffusion, obtained with a "copy from a random neighbor" rule, vs genuine diffusion (b). Both figures are split-screen, starting from a disk, showing half "before" and half "after."

Margolus Algorithm, Lattice gasses

From Toffoli, Cellular Automata Machines, 1988

spread of neutral genes vs diffusion in CA
global vs local particle conservation





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CA as modeling tool: often used generalizations

- probabilistic next state function
  - (deterministic "noisebox")
  - pseudo random generator: SEED
- asynchronous updating

  Note: synchrony implies global control!

  Timescales

  information transfer vs particle conservation
- probabilistic neighbor - choice (within local NB)

can be approximated with "true" CA

Example
CA based models:
not only micro scale and macroscale
MESOscale patterns

Themes:

Setting Baseline expectation

what needs explanation?

Default dynamics vs (evolutionary) selected behavior vs optimal behavior

Lymphnode B-cell nodules

Example: Lymphnodes: B/Tcell nodules

questions: Why this pattern

– *how established*

– *why did the system evolve such that it makes the pattern*

Model

- cross-section
- states: presence/absence of cell B/T at position
- nextstate: birth/death
- influx/outflux of cells (cf particle conservation)
- *Competition for space*

note: clonal selection

The Lymphoid System

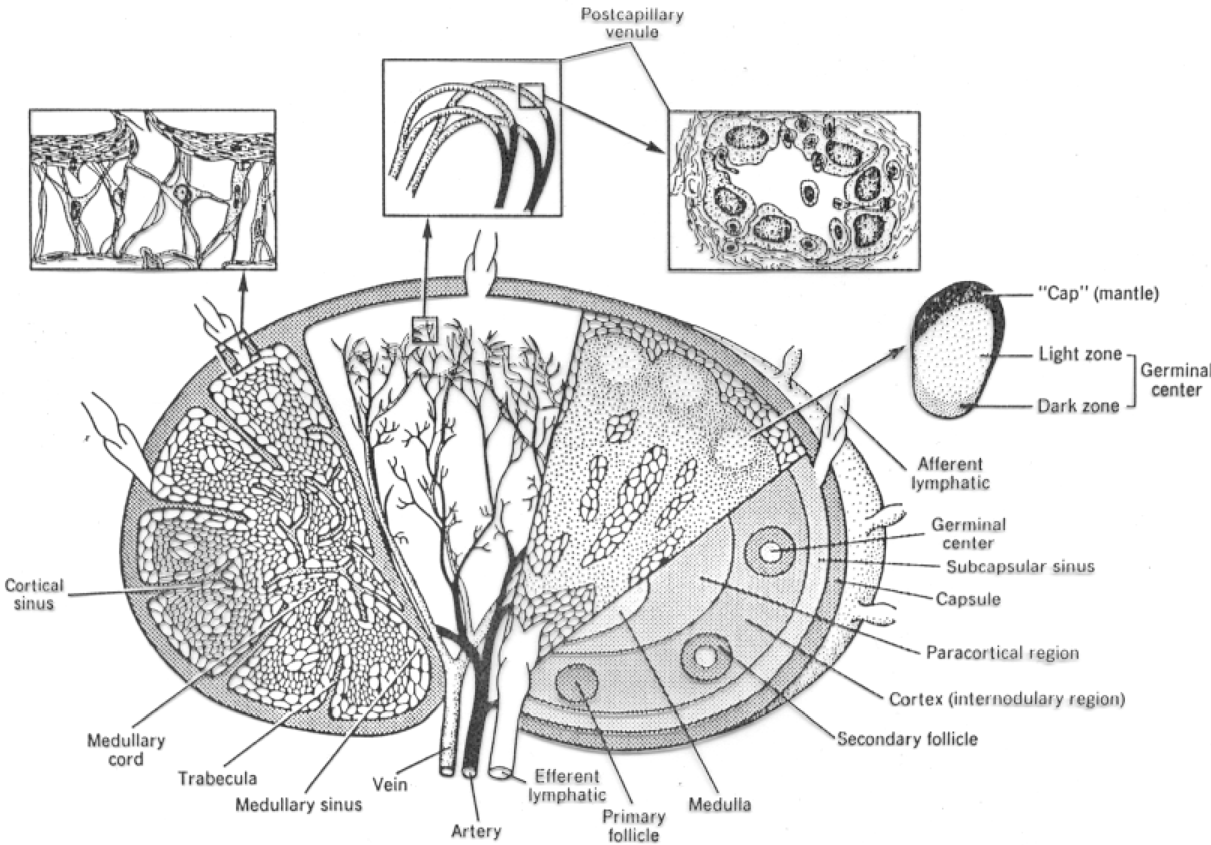


Figure 4.23. Histological organization of a lymph node. The four wedges of the node represent, from left to right, the reticular framework, the circulatory system, the cellular components, and the main structural features in diagram. The four drawings outside the node itself are magnified views of the indicated areas. The third from the left shows the passage of lymphocytes through the postcapillary venule.

The Lymphoid System: CA rules

Next state of empty cell, dependent of number of neighbours of each cell type.

	0	1	2	3	4	5	6	7	8 B cells
0	.	.	.	.	.	.	.	.	.
1	.	B	B	B	B	B	.	.	
2	T	T	T/B	B	B	B	.		
3	T	T	T	T/B	B	.			
4	T	T	T	T	T				
5	T	T	T	.					
6	.	.	.						
7	.	.							
8	.								
T cells									

T: T cell; B: B cell; T/B probability, 0.5 for T cell or B cell.

Next state of empty cell, dependent of number of neighbours of each cell type.

	0	1	2	3	4	5	6	7	8 B cells
0	.	.	.	.	.	.	.	.	.
1	.	T/B	B	B	B	B	.	.	
2	.	T	T/B	B	B	B	.		
3	.	T	T	T/B	B	.			
4	.	T	T	T	T/B				
5	.	T	T	.					
6	.	.	.	.					
7	.	.							
8	.								
T cells									

T: T cell; B: B cell; T/B probability, 0.5 for T cell or B cell.

T/B Cell Segregation

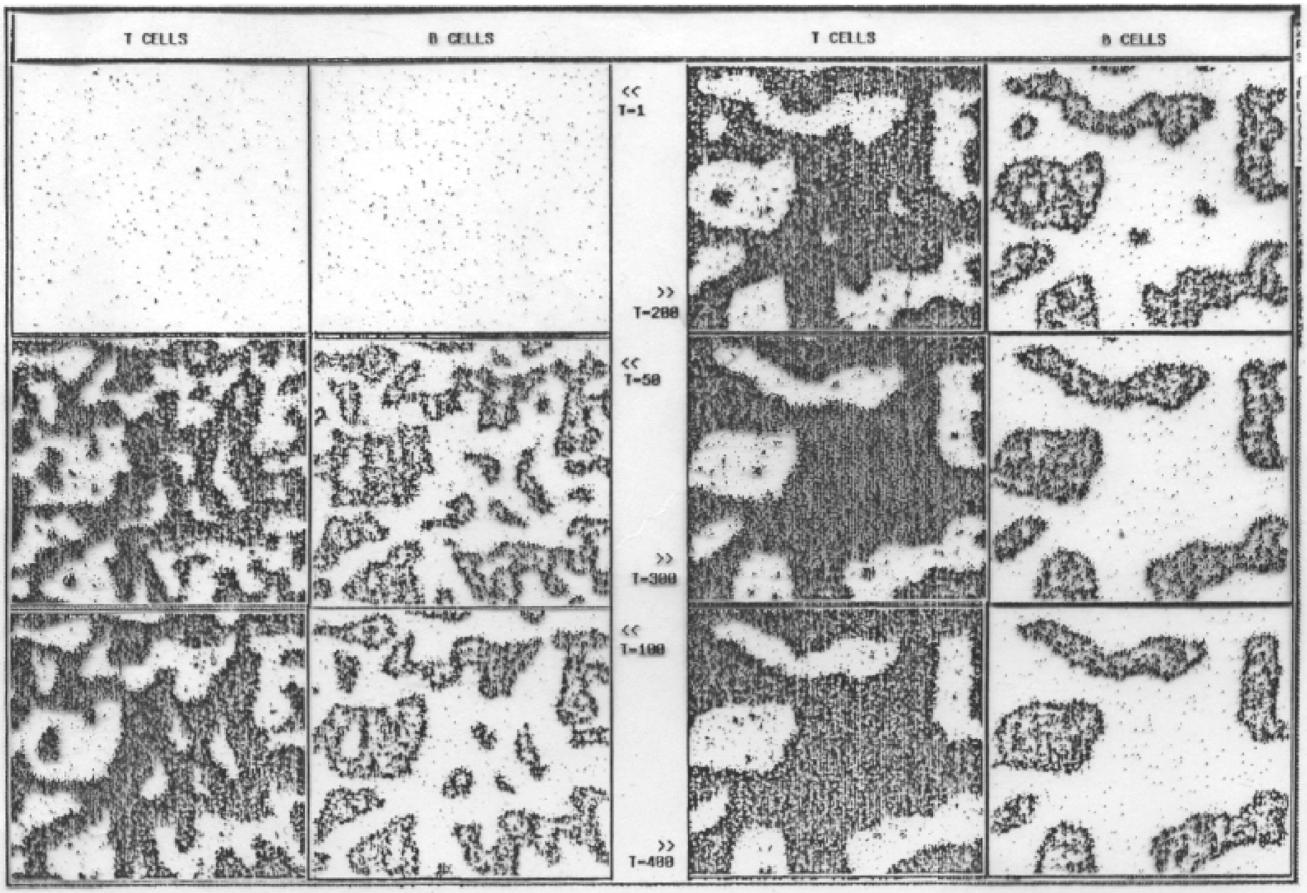


Fig. 2. T/B cell segregation. Large scale patterns develop in cellular automata in which (1) random influx of T cells and B cells occurs (prob 2** – 7 each); (2) both cell types proliferate according to the rules given in Table 1 (i.e. both T cells and B cells need T cells to proliferate into an empty space, and they compete for the empty space); (3) diffusion of cells occurs. Time-steps as indicated.

T/B Cell Expansion

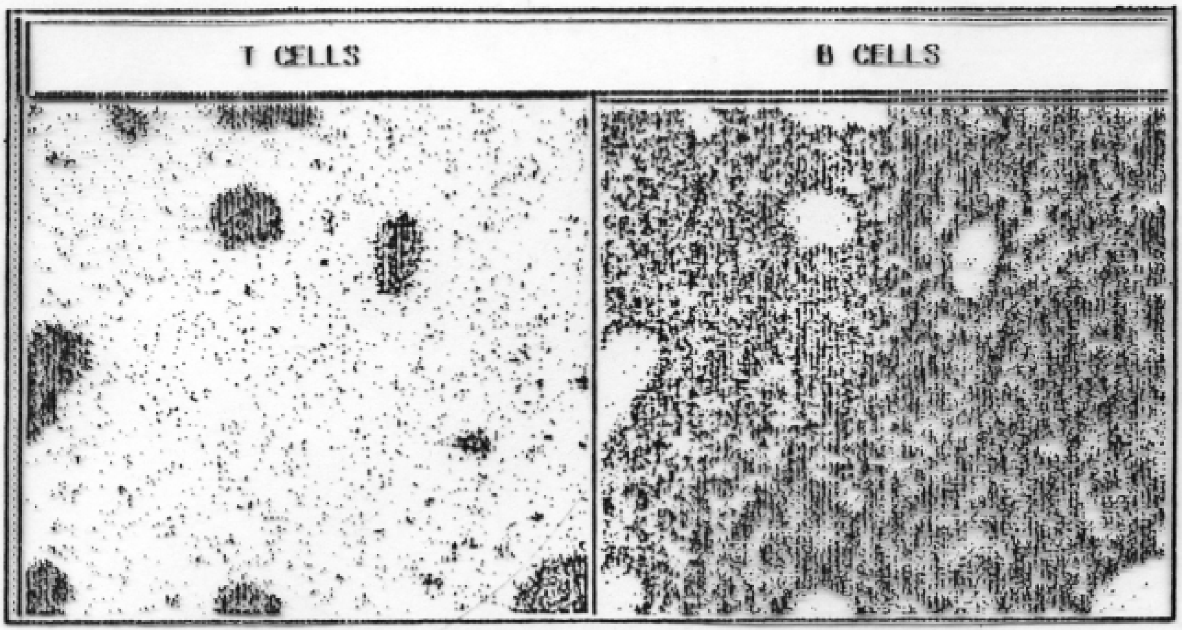
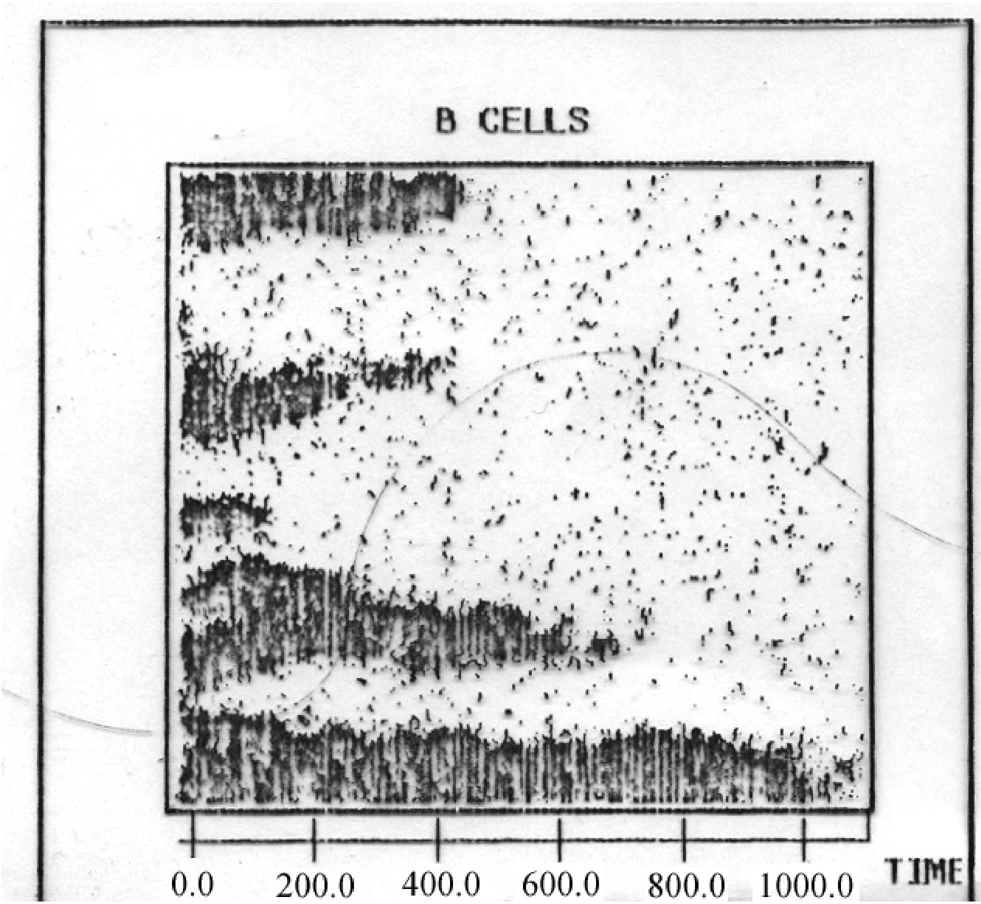


Fig. 3. B cell expansion by increased influx of cells. Identical cellular automaton as in Fig. 2, except that influx of both T and B cells is increased to 2** −6; T = 300.

T/B Cell Expansion



conclusions

Pattern is default (to remain well mixed ’hard’)

’Optimal’ pattern IS well mixed!

“short cut” on full transition table specification modeling formalisms (heuristics) (continued)

Ordering of states (numerical values of variables)

- k Dim state (phase) space (k small)
- transition function (valid for all values)
- *(allows relaxing finiteness)*

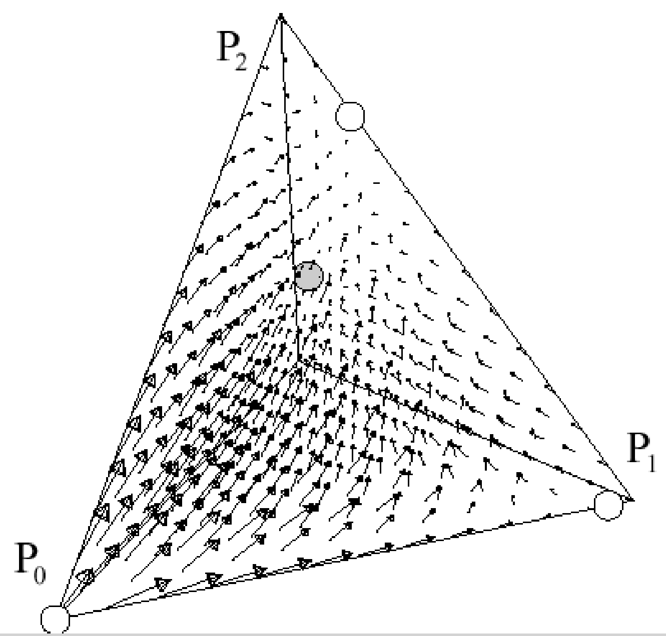
MAP: $\mathbf{X_{t+1} = fX_t}$

ODE : $\mathbf{X' = fX}$

Numerical approximation ODE

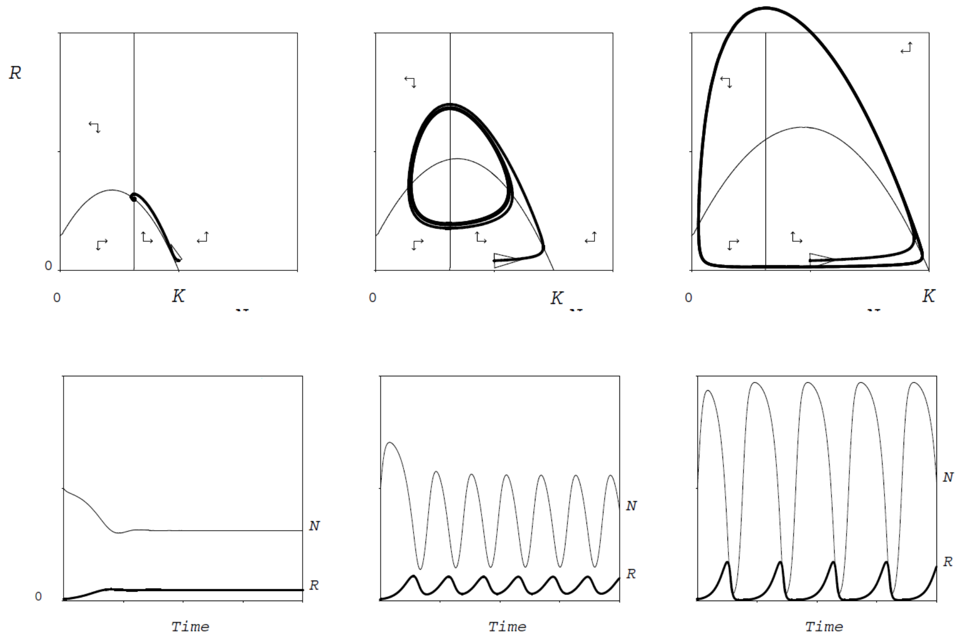
$$\mathbf{X_{t+\Delta t} = X_t + \Delta t * fX_t}$$

“population/concentration”



Studying (nonlinear) ODE (2D): phase-plane analysis (== state space analysis)

Nullclines: set of states for which derivative is zero
Trajectory: set of states visited from initial condition to time=t
Vector field: direction of change at selected states
Attractors: set of states visited - after "enough time"
fixed points; limit cycles; chaotic attractor
Bifurcation diagram: attractors as function of parameter



"ATTO FOX"

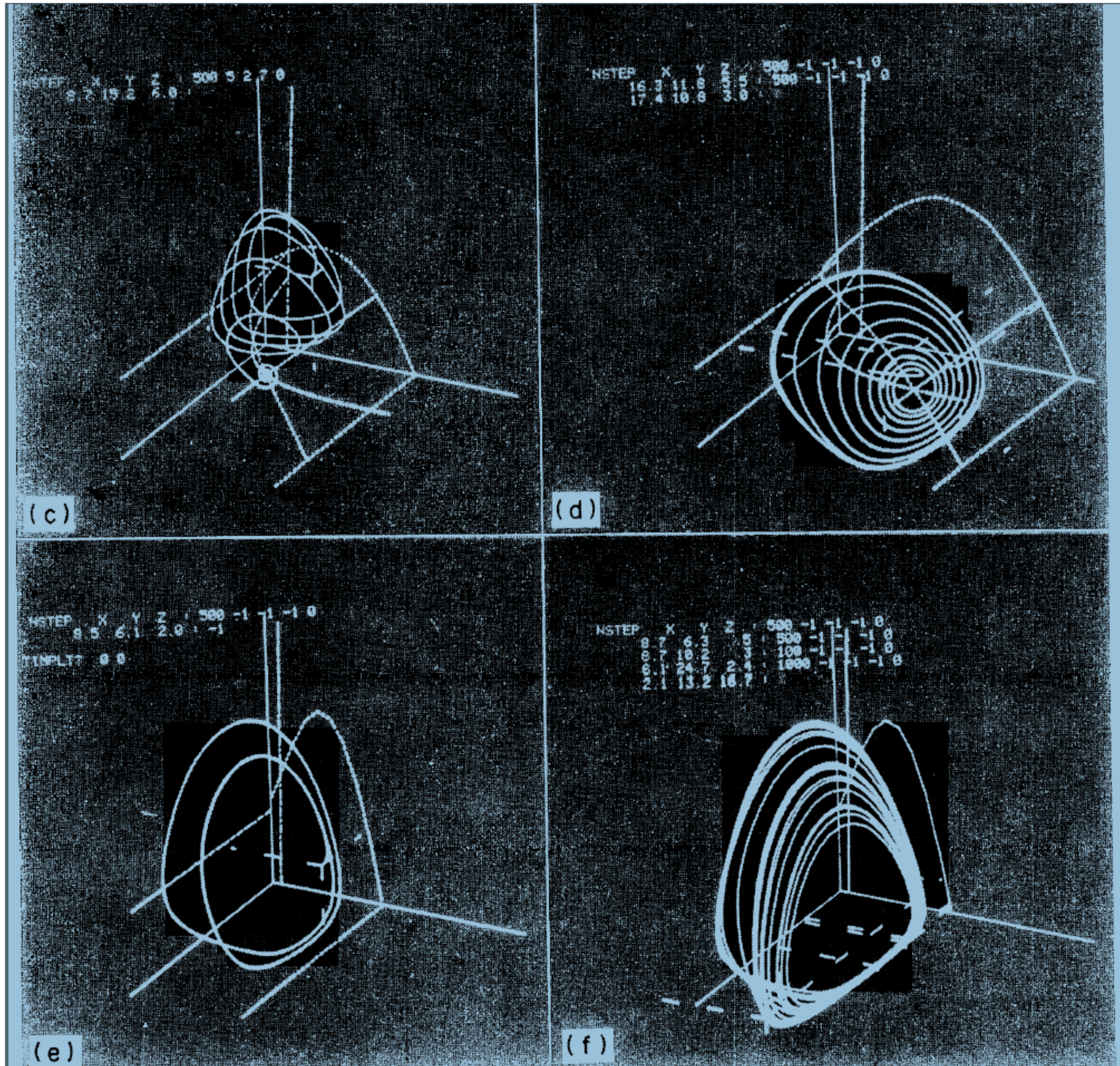
3D

chaotic attractor:

deterministic chaos

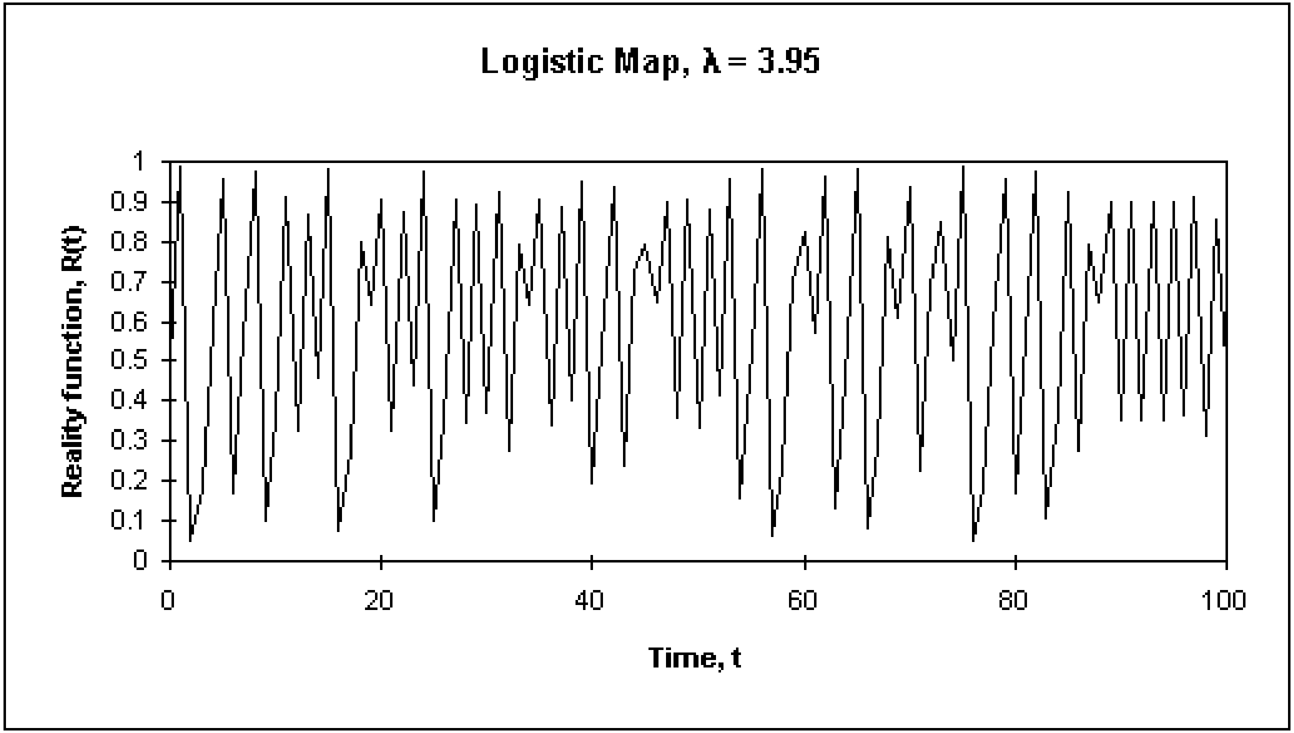
Non-periodic

(period doubling)



MAPS: also deterministic chaos in 1D maps

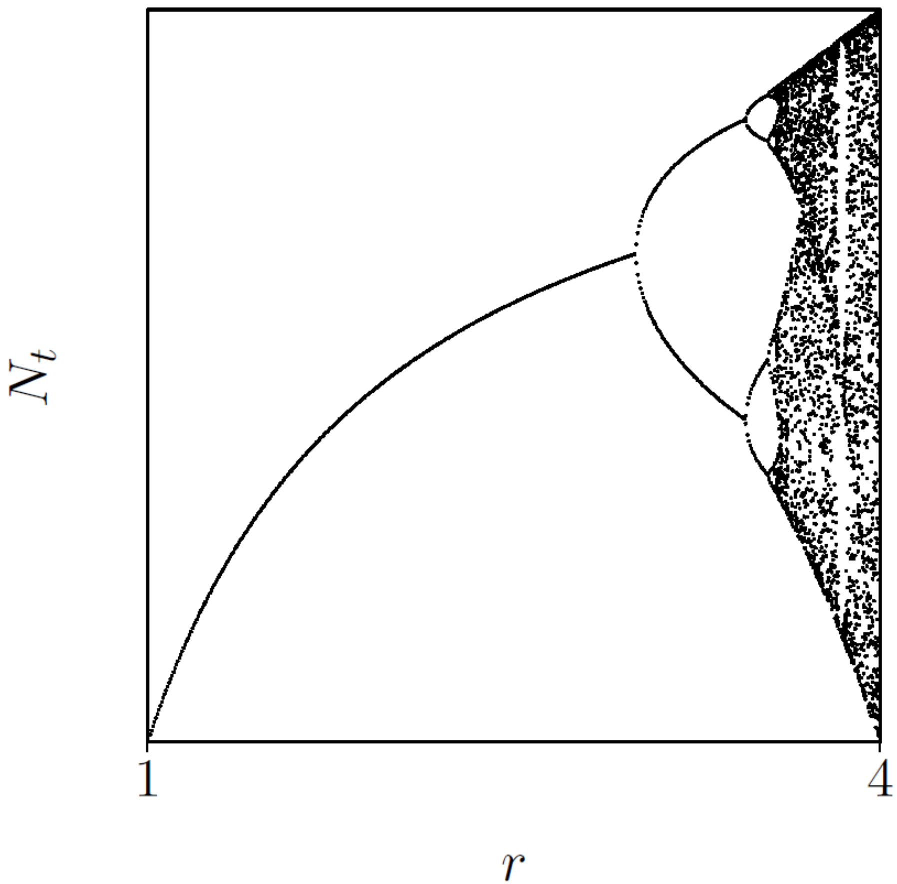
best known example: logistic map $N_{t+1} = \lambda N_t(1 - N_t)$

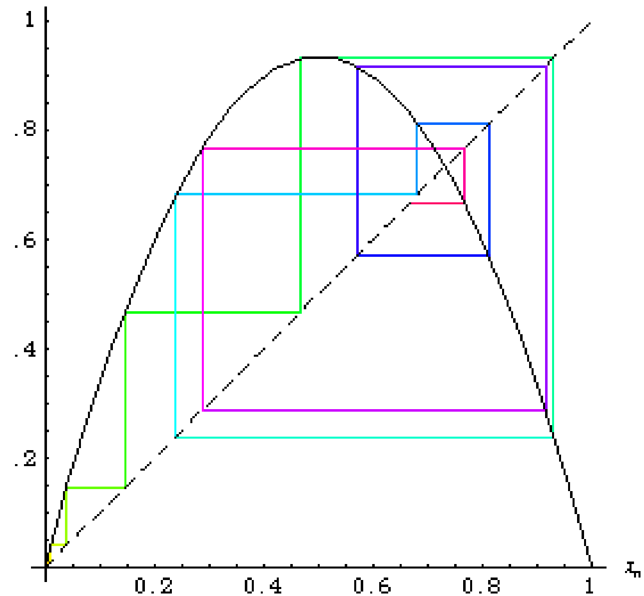


MAPS: also deterministic chaos in 1D maps

best known example: logistic map $N_{t+1} = rN_t(1 - N_t)$

bifurcation diagram, Takens plot, cobweb



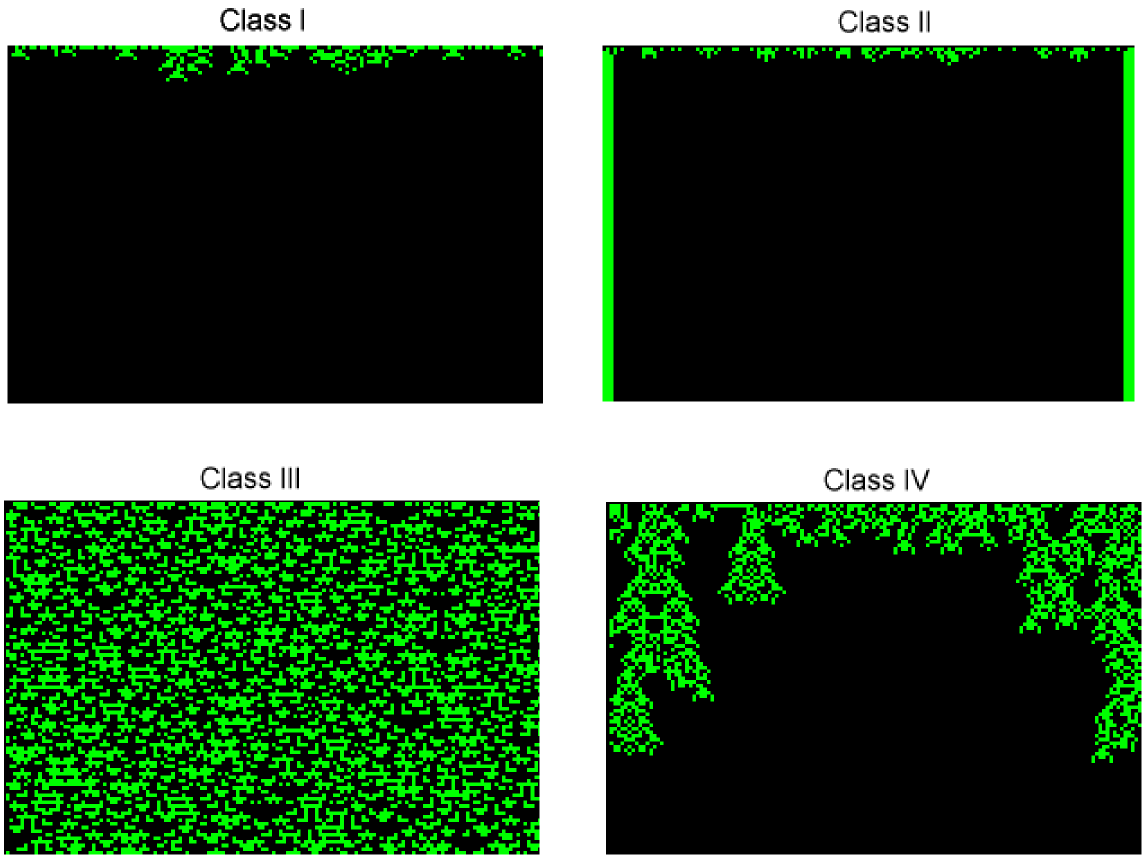


(autonomous) Dynamical systems: basic properties

- unique nextstate function (cf vector field)
- attractors: fixed point, limit cycle, chaotic attractor
- basin of attraction
- transient

manifestation in CA

CA classification (Wolfram)
eyeball spacetimeplots of (1D) CA's ,
random initial conditions

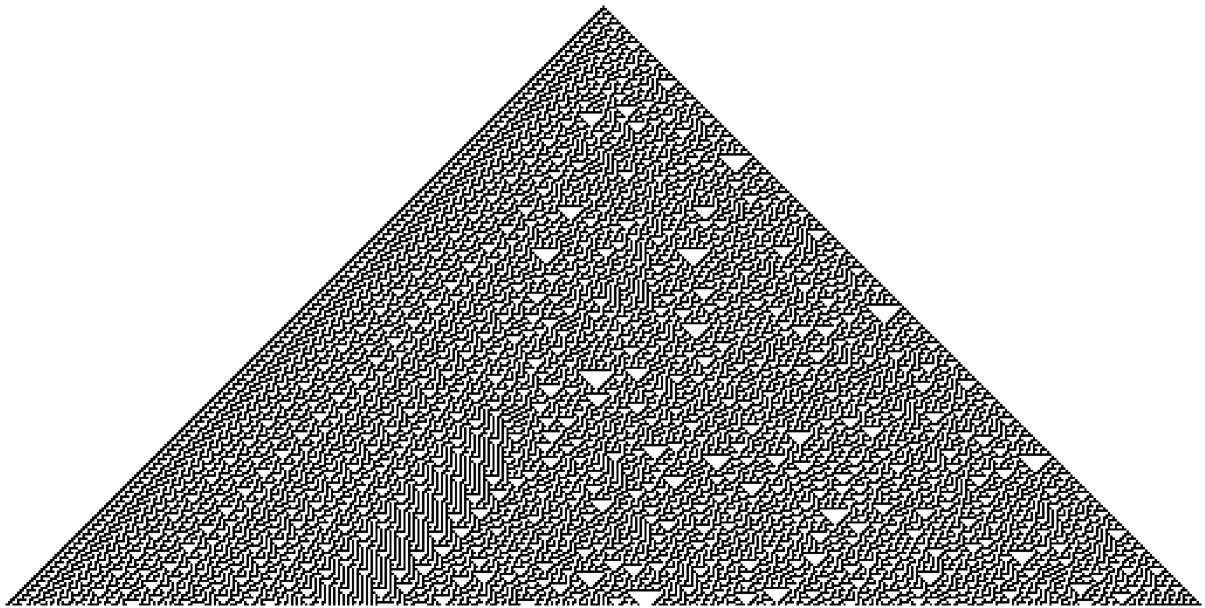


High dimensional chaos

~ random   # of 1 ~ constant

Universal computation

...from single non-quiescent state



Rule 30 cellular automaton

Classification of CA's (Wolfram, Langton)

	spatial pattern	non-spatial analogue
Class I :	to uniform state	fixed point
Class II a	domains, localized	limit cycles
Class II b	idem non-stationary	idem
III	non(>>)-periodic , non-localized	chaos (high dim.)
Class IV	loc. + non-loc., long transient	universal computation

Order parameter $\lambda =$ Fraction of rules to the non-quiescent

state

I——IIa ——IIb-IV-III——

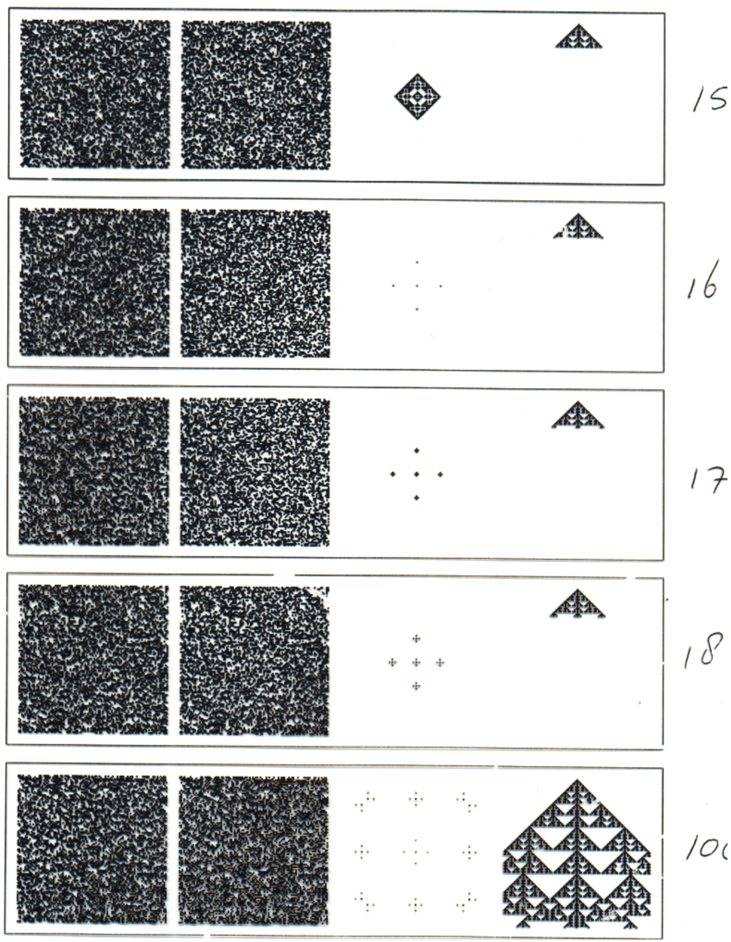
viz Modulo Prime, Game of Life, Voting

Almost all cases

“Irrducible computation”

high dimensinal chaos and random noise

Modulo Prime: ($\lambda = .5$) type 3 chaos
random IC, one bitflip difference



Comparing/combining modeling approaches
Model entities, model observables, modeltransformations (1)

CA and/or ODE

Example: Modeling birth/death processes
Dynamical system: fixed set of states/variables,interactions

How to model a variable set of individuals?

- In CA (discrete space/time):
  fixed set of automata (patches)
  individuals as state of patch of space.
  birth: s=0 (empty square) copies state of a nb
  death: s=1 (occupied by idividual) $->$ s=0 (with prob d)
  'population' as observable.

- in ODE (continuous time/variables): (e.g. $dN/dt = aN - bN^2$)
  MAPS (discrete time, cont variables): (e.g. $N_{t+1} = (a+1)N_t - bN_t^2$)

  fixed set of variables (here 1)
  birth/death changes in values of variables.
  'population' is model entity AND observable.

relating CA en ODE models: ODE as MEAN FIELD 'APPROXIMATION' of CA

$$dN/dt = aNE/T - dN$$

$$dN/dt = aN(1 - N) - dN$$

$$dN/dt = (a - d)N - aN^2$$

E is empty space

-'simplification' to population based description
- mixing (localness vs pattern formation)
- NOTE: lumping/naming of parameters.

VS Mean Field Assumption(!)

Overview single level (Autonomous) Dynamical Systems
timing regimes

	continuous time	discrete time
continuous var.	ODE	MAPS
discrete var./ nominal entities	??	**FSM** *n-FSMs: CAs, B-nets*

Overview single level (Autonomous) Dynamical Systems
timing regimes

	continuous time	discrete time
continuous var.	ODE	MAPS
discrete var./ nominal entities	EVENT	**FSM** *n-FSMs: CAs, B-nets*

event based modeling and multiple timescales: event scheduling

Event based modeling: “something changes sometimes”

Global, population based: Gillespie algorithm:
calulate when next event happens
determine which event from relative probabilities.

Local, individual based algorithm: time-line
Actions take time. Next action scheduled at time+dt
Muliple timescales can occur at minimal computational cost.

EVENT based models: continuous time, discrete events
Gillespie algorithm: simple example

1: seen als stochastic ODE

Example: logistic stochastic population growth

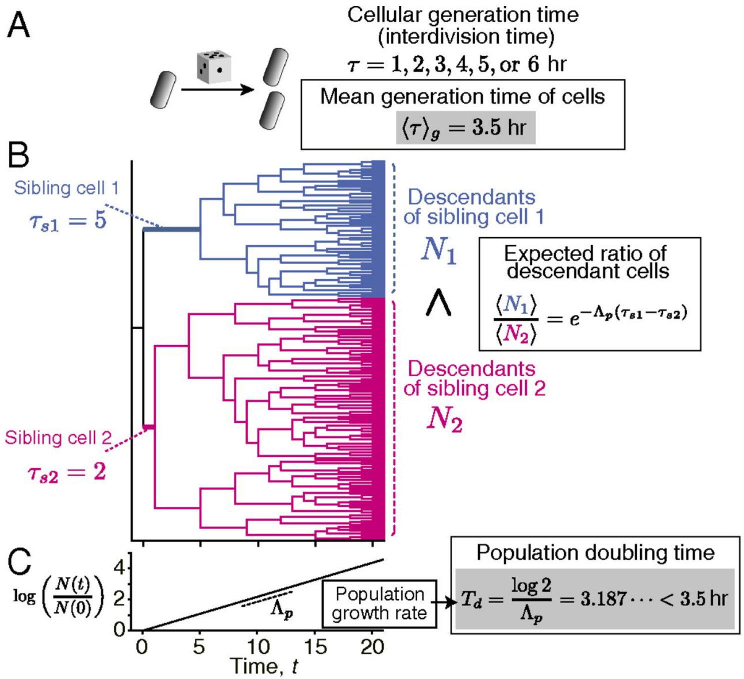
$$dN/dt = aN - bN^2 + noise$$

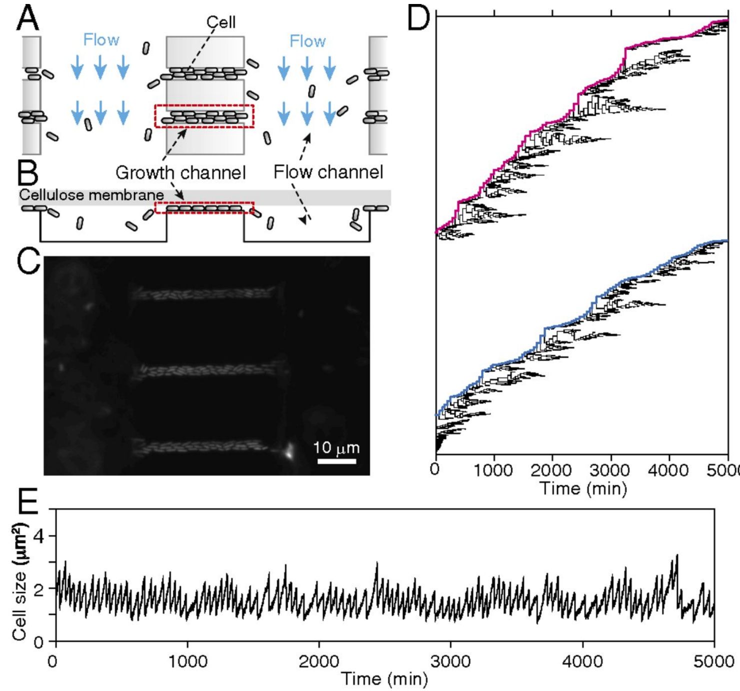
EVENT based

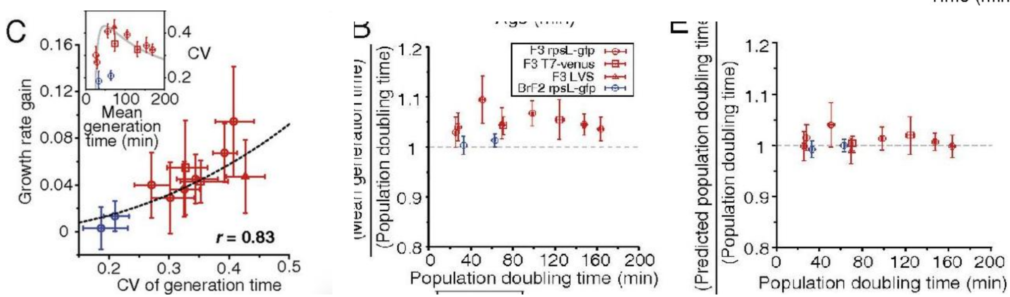
all events (birth + death) :
$e_0 = (a_1 + a_2)N - b_1 N^2 + b_2 N^2$
$\tau = 1/e_0 ln(1/rand1)$; $T = T + \tau$
N=N+1 if $(a_1 N - b_1 N^2) < rand2 * e_0$
else N=N-1;

Population vs Individual doubling time
population grows faster than average individual

(Hashimoto et al PNAS 2016 113 (12) 3251-3256)







Conclusions

Modeling formalisms: CA, ODE, MAPS, EVENTS

Individual based models vs population based models

Example

Simple growth process: same basic assumptions

DIFFERENT behaviour.

Population of "average" individuals $= / =$ sum of individuals.

In tutorials also seen in CA

Models as simple, but what is simple is different