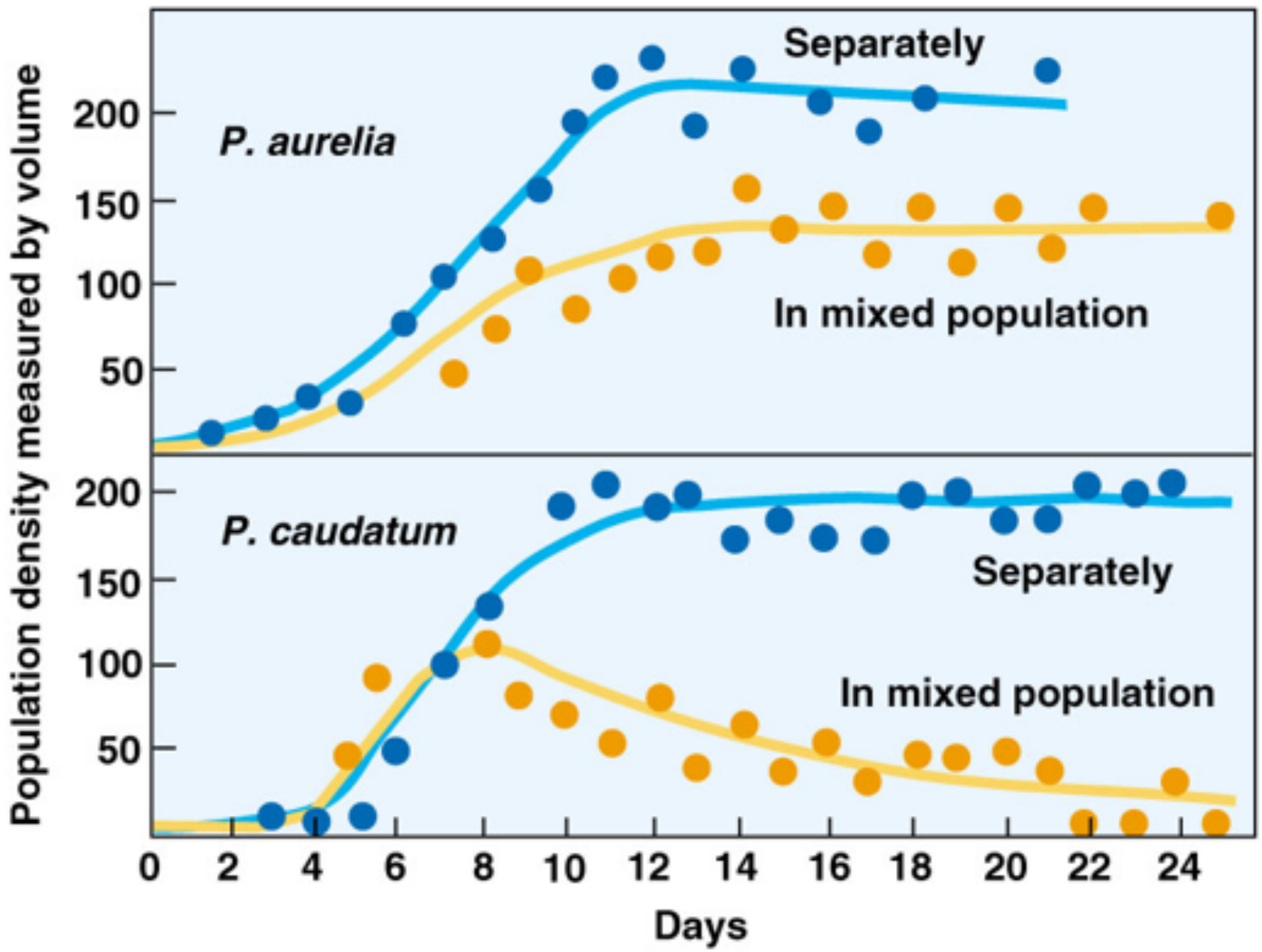
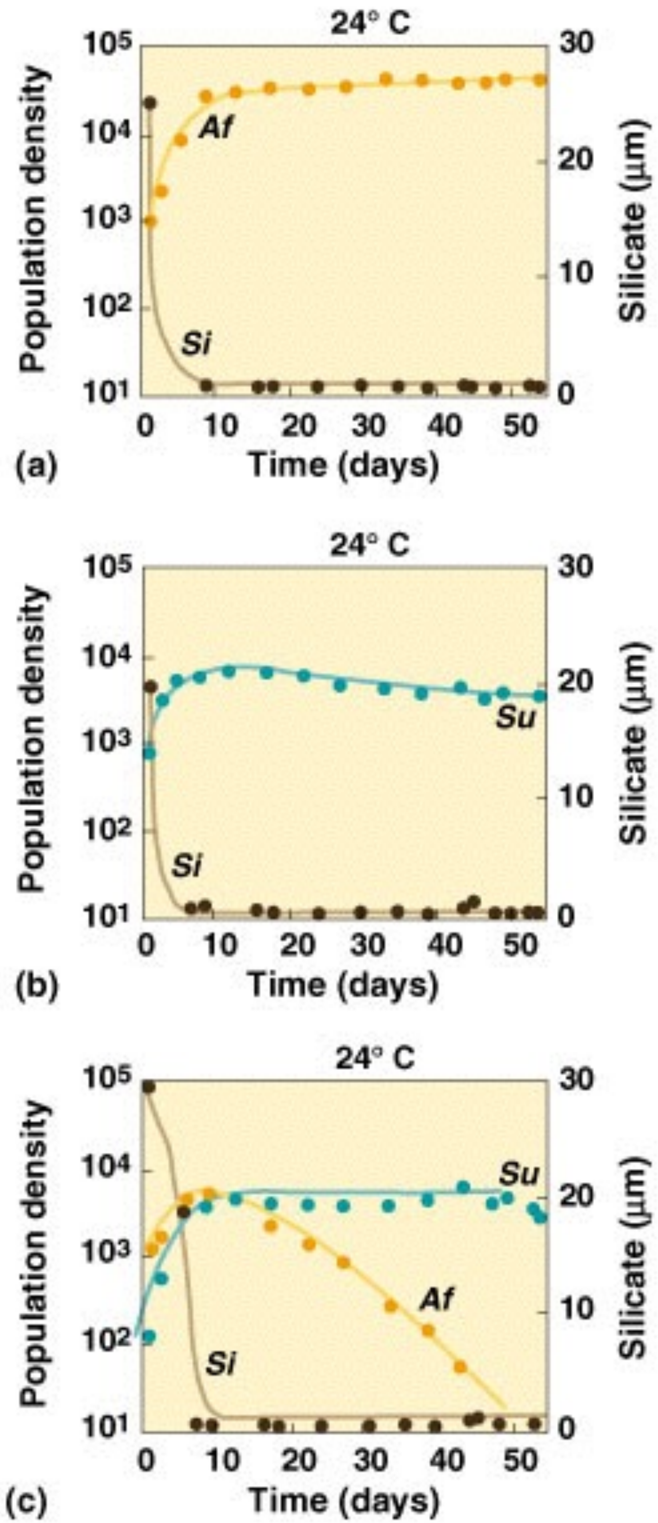
Chapter 5: Competitive exclusion



Gause: Paramecium (1934)

Theoretical Biology 2015



Competitive exclusion
Tilman 1981

Two species of diatom feeding on silica:
Af: *Asterionella formosa*
Su: *Synedra ulna*, Si: silica

From: Smith & Smith, Elements of Ecology

Question 6.5 on immune responses

$$\frac{\mathrm{d}T}{\mathrm{d}t} = \sigma - \delta_T T - \beta T I \ , \qquad \frac{\mathrm{d}I}{\mathrm{d}t} = \beta T I - \delta_I I - k_1 I E_1 - k_2 I E_2 \ ,$$

$$\frac{\mathrm{d}E_1}{\mathrm{d}t} = \alpha_1 E_1 I - \delta_E E_1 \quad \text{and} \quad \frac{\mathrm{d}E_2}{\mathrm{d}t} = \alpha_2 E_2 I - \delta_E E_2 \ .$$

dE./dt gives: $I=\delta_E/\alpha_1$ and $I=\delta_E/\alpha_2$.

E_1 and E_2 have to be solved from dI/dt=0.

Substitute $I=\delta_E/\alpha_1$ into dE_2/dt:

$$\frac{\mathrm{d}E_2}{\mathrm{d}t} = \delta_E E_2 \left(\alpha_2/\alpha_1 - 1\right) < 0$$

Simplest mathematical model

Resource (e.g., amount of nitrogen available):

$$R = 1 - N_1 - N_2$$

Two consumers

$$\frac{dN_1}{dt} = N_1(b_1 R - d_1) \quad \text{and} \quad \frac{dN_2}{dt} = N_2(b_2 R - d_2)$$

With a fitness of

$$R_{0_1} = b_1/d_1 \quad \text{and} \quad R_{0_2} = b_2/d_2$$

Nullclines

Substitution of $R = 1 - N_1 - N_2$ into

$$\frac{\mathrm{d}N_1}{\mathrm{d}t} = N_1(b_1 R - d_1)$$

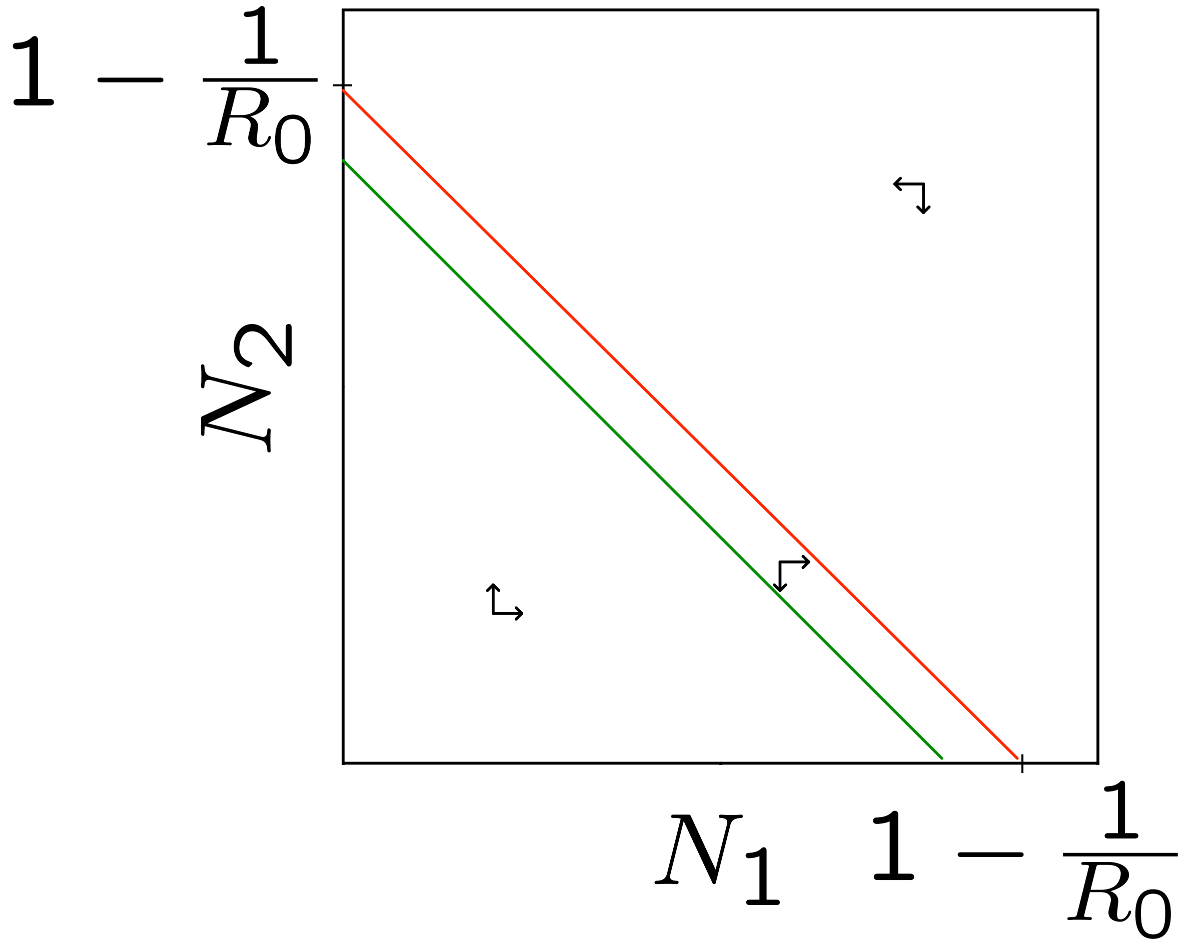
yields

$$\frac{\mathrm{d}N_1}{\mathrm{d}t} = N_1\left(b_1(1 - N_1 - N_2) - d_1\right)$$

which has nullclines $N_1 = 0$ and

$$N_2 = 1 - \frac{1}{R_{0_1}} - N_1$$

Nullclines



Similarly the $\mathrm{d}N_2/\mathrm{d}t = 0$ nullcline is found to be

$$N_2 = 1 - \frac{1}{R_{0_2}} - N_1$$

Thus, plotting N_2 as a function of N_1 the two nullclines run parallel with slope -1.

Paradox of the plankton, bacteria in the gut, ...

How can so many species co-exist?
co-existence not an equilibrium?
Space is not homogeneous?
Species are so similar that exclusion is slow?
Species are largely controlled by parasites?