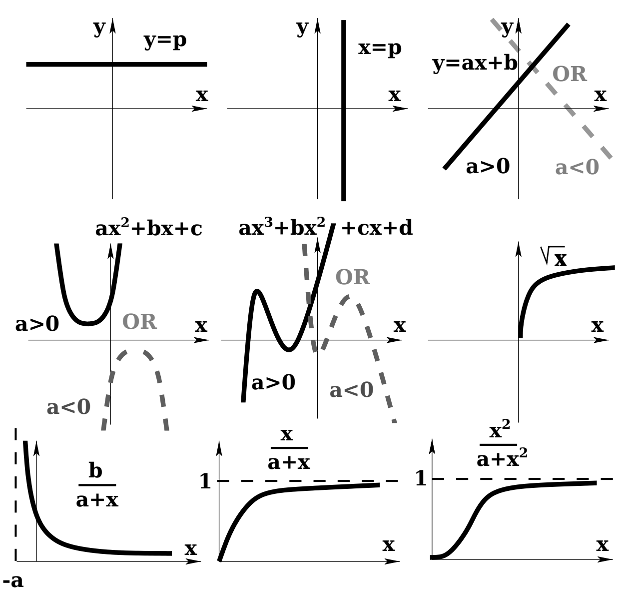
Common graphs:



Quadratic equation: The general solution of a quadratic equation $ax^2 + bx + c = 0$ is given by the so-called *abc-formula*:

$$x_{1,2} = \frac{-b \pm \sqrt{D}}{2a} \quad \text{with} \quad D = b^2 - 4ac \ , \quad \text{and}$$

complex numbers are obtained when $D < 0$, by defining $\mathrm{i}^2 = -1 \Leftrightarrow \mathrm{i} = \sqrt{-1}$, e.g., $\sqrt{-2} = \mathrm{i}\sqrt{2}$.

Linearization:
$f(x,y) \simeq f(\bar{x},\bar{y}) + \partial_x f(\bar{x},\bar{y})\,(x-\bar{x}) + \partial_y f(\bar{x},\bar{y})\,(y-\bar{y})$

The 1D linear differential equation $\mathrm{d}N/\mathrm{d}t = kN$ has the solution: $N(t) = N_0 \mathrm{e}^{kt}$, where N_0 is an (arbitrary) initial value of N.

The solution of a linear system of ODEs

$$\begin{cases} \mathrm{d}x/\mathrm{d}t = ax + by \\ \mathrm{d}y/\mathrm{d}t = cx + dy \end{cases} \leftrightarrow \begin{pmatrix} \mathrm{d}x/\mathrm{d}t \\ \mathrm{d}y/\mathrm{d}t \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

comes from the characteristic equation: $\lambda^2 - \mathrm{tr}\lambda + \det = 0$, where $\mathrm{tr} = a + d$ and $\det = ad - bc$, i.e., $\lambda_{1,2} = (\mathrm{tr} \pm \sqrt{D})/2$, where $D = \mathrm{tr}^2 - 4\det$. When $D > 0$ the eigenvalues are real, otherwise they form a complex pair $\lambda_{1,2} = \alpha \pm \mathrm{i}\beta$, where $\alpha = \mathrm{tr}/2$ and $\beta = \sqrt{-D}/2$. The general solution is given by

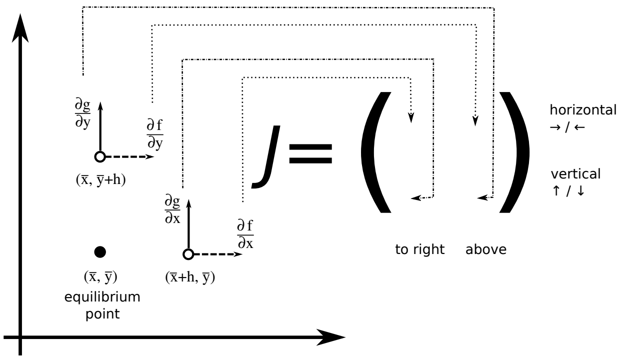
$$\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = C_1 \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} \mathrm{e}^{\lambda_1 t} + C_2 \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} \mathrm{e}^{\lambda_2 t} \ ,$$

which grows whenever $\lambda_{1,2} > 0$. The eigenvectors are found by substituting λ_1 and λ_2 into:

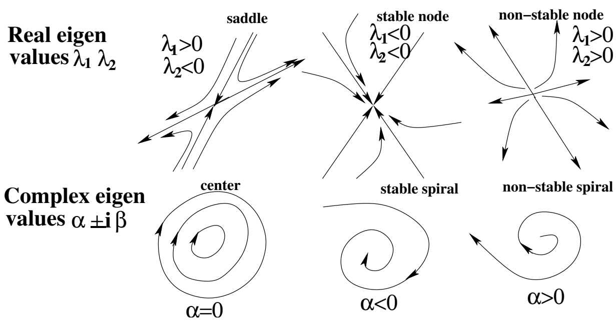
$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -b \\ a - \lambda_i \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} d - \lambda_i \\ -c \end{pmatrix}$$

For **general non-linear systems** $\begin{cases} \mathrm{d}x/\mathrm{d}t = f(x,y) \\ \mathrm{d}y/\mathrm{d}t = g(x,y) \end{cases}$ the equilibria are solved from setting $f(x,y) = 0$ and $g(x,y) = 0$. The $x' = 0$ and $y' = 0$ nullclines are given by $f(x,y) = 0$ and $g(x,y) = 0$, respectively. The vector field switches at the nullclines, and can be determined from an extreme value of x and/or y. The equilibrium type can be found by linearizing the ODEs and evaluating the trace and determinant of the **Jacobian** $J = \begin{pmatrix} \partial_x f & \partial_y f \\ \partial_x g & \partial_y g \end{pmatrix}$ at the equilibrium.

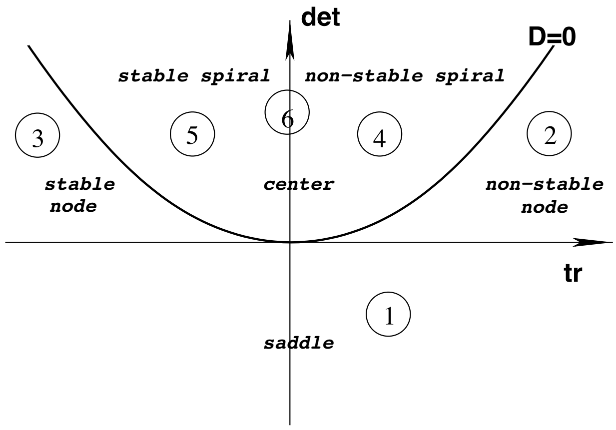
The signs $(+, -, 0)$ of these partial derivatives can be determined using the **graphical Jacobian** method:



Eigenvalues determine the **equilibrium type**, as shown in the figure below, where the straight lines are the eigenvectors:



The equilibrium type can be determined form the trace and determinant of the Jacobian:



Common equations:

Equation	Solution	Conditions
$x^n = p$	$x = p^{\frac{1}{n}} = \sqrt[n]{p}$	$x > 0,\ p > 0$
$g^x = c$	$x = \log_g c$	$x > 0,\ g > 0,\ g \neq 1$
$\log_g x = b$	$x = g^b$	$g > 0,\ g \neq 1$
$e^x = c$	$x = \ln c$	$c > 0$
$\ln x = b$	$x = e^b$	

Working with powers

$a^0 = 1$	$a^{-1} = \frac{1}{a}$	$a^{\frac{1}{2}} = \sqrt{a}$	$a^{\frac{p}{q}} = (\sqrt[q]{a})^p$
$a^1 = a$	$a^{-p} = \frac{1}{a^p}$	$a^{\frac{1}{q}} = \sqrt[q]{a}$	$a^{-\frac{p}{q}} = \frac{1}{(\sqrt[q]{a})^p}$
$0^p = 0$			

$a^p \times a^q = a^{p+q}$	$\frac{a^p}{b^q} = a^p \times b^{-q}$	$(a \times b)^p = a^p \times b^p$
$\frac{a^p}{a^q} = a^p \times a^{-q} = a^{p-q}$	$(a^p)^q = a^{pq}$	$(a^p \times b^q)^r = (a^p)^r \times (b^q)^r = a^{pr} \cdot b^{qr}$

Working with fractions

$\frac{a}{b} = \frac{ca}{cb}$	$\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$	$\frac{a}{\frac{b}{c}} = a \times \frac{c}{b} = \frac{ac}{b}$
$\frac{a}{b} \times c = \frac{ca}{b}$	$\frac{ac}{bd} = a \times c \times \frac{1}{b} \times \frac{1}{d} = \frac{a}{b} \times \frac{c}{d}$	$\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$

Logarithms

The following applies: if $x = n^b$, then $\log_n x = b$, with $n > 0$ and $n \neq 1$. For instance, $\log_{10} x$ tells you to what power you should raise 10 (so how many times you should multiply 10 with itself) to get the number x. The following rules apply to working with logarithms, provided $a, b, n, q > 0$ and $n, q \neq 1$:

$\log = \log_{10}$	$\log_n ab = \log_n a + \log_n b$	$\log_n a^p = p \times \log_n a$
$\ln = \log_e$	$\log_n \frac{a}{b} = \log_n a - \log_n b$	$\log_n a = \frac{\log_q a}{\log_q n}$

Derivatives

	function	**derivative**
	$g(x) = cf(x)$	$g'(x) = cf'(x)$
sum rule	$p(x) = f(x) + g(x)$	$p'(x) = f'(x) + g'(x)$
product rule	$q(x) = f(x)g(x)$	$q'(x) = f'(x)g(x) + f(x)g'(x).$
chain rule	$r(x) = f(g(x))$	$r'(x) = f'(g(x))g'(x)$
quotient rule	$q(x) = \frac{f(x)}{g(x)}$	$q'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$

Derivatives for some common functions:

$x^n \to nx^{n-1}$	$\log_n x \to \frac{1}{x \ln n}$
$e^x \to e^x$	$\sin x \to \cos x$
$g^x \to g^x \ln g$	$\cos x \to -\sin x$
$\ln x \to \frac{1}{x}$	$\tan x \to \frac{1}{\cos^2 x} = 1 + \tan^2 x$

Complex numbers:

The addition of complex numbers is adding their real and imaginary parts, $(a+b\mathrm{i})+(c+d\mathrm{i}) = (a+c+[b+d]\mathrm{i})$, like summing vectors. The multiplication of complex numbers follows similar rules:

$$(a + b\mathrm{i})(c + d\mathrm{i}) = (ac + ad\mathrm{i} + bc\mathrm{i} + bd\mathrm{i}^2) = (ac - bd + [ad + bc]\mathrm{i}).$$